(Pages : 4) **6672**

Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, November 2013
(2008 Scheme)
08.502 : ADVANCED MATHEMATICS AND QUEUING MODELS (R F)

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions of Part-**A** and **one full** question **each** from Module **I**, Module **II** and Module **III** of Part-**B**.*

PART – A

1. Define basic solution, basic feasible solution, degenerate and non-degenerate solutions of a linear programming problem.

2. Explain the role of artificial variables in LPP. Give example.

3. Define the following connected with PERT Network. Optimistic time, Most likely time, Pessimistic time and Expected time of an activity.

4. Construct a network diagram based on the following information

Activity	:	A	B	C	D	E	F	G	H
Proceeded by	:	–	–	A	B	C, D	D	E	F

5. Show that the set of all 2×2 non-singular matrices is not a vector space. Show also that the set of singular matrices is not a vector space.

6. Prove that (1, 0, 1, 0), (0, 1, 0, 1) and (0, 0, 0, 2) in $\mathbb{R}^4$ are linearly independent over $\mathbb{R}$.

7. Show that the quadratic form $x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2xz$ is indefinite.

8. What are the basic characteristics of a queueing model ?

9. If λ and μ are the average number of arrivals and average number of departures respectively of an $(M|M|1):(\infty|FIFO)$ queueing model and $P(N > k)$ denote the probability that the number of customers in the system exceeds k, then show that $P(N > k) = \rho^{K+1}$ where $\rho = \frac{\lambda}{\mu}$.

10. Explain $(M|M|C):(\infty|FIFO)$ queueing model. **(10×4=40 Marks)**

PART – B

Module – I

Answer **one full** question.

11. a) Solve by simplex method

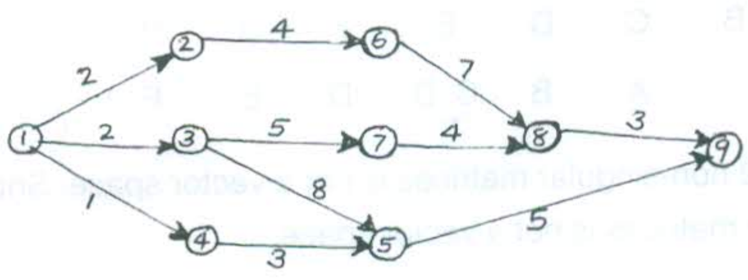
Minimize $Z = 5x_1 + 6x_2$

Subject to $2x_1 + 5x_2 \geq 1500$

$3x_1 + x_2 \geq 1200$

$x_1, x_2 \geq 0.$ 10

b) Find the critical path and project duration of the following network 10



12. a) Solve by simplex method

Minimize $Z = 3x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \geq 90$

$4x_1 + 3x_2 \geq 120$

$x_1, x_2 \geq 0.$ 10

b) A small project is composed of eight activities whose time estimates are given below

Activity	1 – 2	1 – 3	2 – 4	2 – 5	3 – 5	4 – 5	4 – 6	5 – 6
Pessimistic	21	27	8	3.5	10	1.7	9	5
Most likely	7.5	8	8	2	10	1	7.5	3
Optimistic	3	3	8	0.5	10	0.3	3	1

i) Draw the network and identify the critical path.

ii) Find the probability of completing the project no more than 4 weeks later than expected. 10

Module – II

13. a) Solve the following equations AX = B using LU factorization of A.
$x + 2y + z = 1;\ 2x + 6y + 7z = 3;\ x + 5y + 8z = 2.$ 10

b) Express $v = (2, -5, 4)$ in $\mathbb{R}^3$ as a linear combination of the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$ and $u_3 = (2, 3, 8)$. 10

14. a) Find a basis for the null space of $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix}$. 7

b) Find the orthogonal complement of the plane spanned by the vectors (1, 1, 2) and (1, 2, 3). 7

c) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$. 6

Module – III

15. a) Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes.

i) What is the probability that a person arriving at the booth will have to wait ?

ii) What is the average length of the queue that will form from time to time ?

iii) The telephone department will install a second booth when convinced that an arrival will be expected to wait for at least 3 minutes for the phone. By how much should the flow of arrivals increase in order to justify a second booth ? 10

b) A fertilizer company distributes its products by trucks loaded at its only loading station. Both company trucks and contractor's trucks are used for this purpose. It was found that on an average every 5 minutes one truck arrived and the average loading time was 3 minutes. 40% of the trucks belong to the contractors. Making suitable assumptions, determine

i) The probability that a truck has to wait.

ii) The expected waiting time of a truck.

iii) The expected waiting time of contractor's trucks per day. **10**

16. A petrol pump station has 4 pumps. The service time follows the exponential distribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

a) What is the probability that an arrival would have to wait in line ?

b) Find the average waiting time, average time spend in the system and the average number of cars in the system.

c) For what percentage of time would a pump be idle on an average ? **20**
